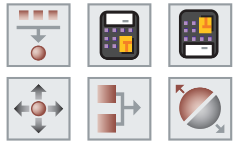
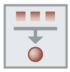
3 Basic Computations



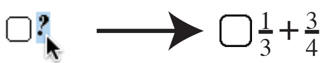
Let's start doing some math! This chapter is about baby steps and learning the basics of computations. We are just starting the LiveMath engine here. Look for future chapters on more powerful computations.

3.1 Simplify 

In a nutshell, the Simplify command *cleans things up*.

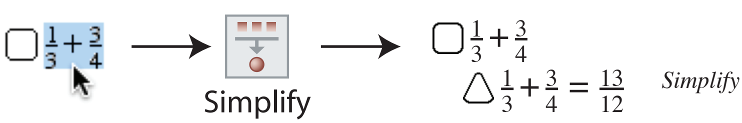
What does this mean mathematically? Well, it means a number of things, all of which we will explain in the course of this section. Being good mathematicians, let's start off with some experiments.

Let's enter in a basic computation into a math computation **Assumption**  like this:



There are a few different ways to enter this expression. From the keyboard is the easiest:

Now we select the expression by double-clicking on the + sign (the major operation in the expression), and click on the Simplify  icon in the Palette:



Notice the answer format: the result is a *fraction*. That is in keeping with Simplify's mantra: *clean things up* - do the calculations and combine similar objects together into similar objects, and stay symbolic, if you can.

Notice that *LiveMath* does not change such a fraction to a "mixed fraction", like "1 $\frac{1}{12}$" as is common in grammar through high-school type math. Some teachers from below university levels have asked for this feature for their instructional purposes. The problem is that technically speaking, "1 $\frac{1}{12}$" means something completely different in *LiveMath*: 1 times $\frac{1}{12}$, which is not what you want at all.

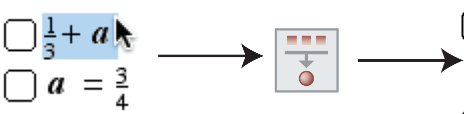
If we add one fraction to one decimal number, Simplify will choose the easier of the two formats – decimals – and give the answer in decimal format:

$$\square\ \frac{1}{3}+0.75$$
$$\triangle\ \frac{1}{3}+0.75 = 1.0833 \quad \textit{Simplify}$$

Even though this answer could be described by a nice fraction, *LiveMath* does not try to change it to a fraction, per se. If you do want this final answer as a fraction, you can try the Uncalculate  command:

$$\square\ \frac{1}{3}+0.75$$
$$\triangle\ \frac{1}{3}+0.75 = 1.0833 \quad \textit{Simplify}$$
$$\triangle\ \frac{1}{3}+0.75 = \frac{13}{12} \quad \textit{UnCalculate}$$

Simplify does not go looking around for values for substitution. Simplify is more polite, and will just try to clean up your expressions algebraically (or numerically):



$$\square\ \frac{1}{3}+a$$
$$\triangle\ \frac{1}{3}+a = a+\frac{1}{3} \quad \textit{Simplify}$$
$$\square\ a = \frac{3}{4}$$

The Assumption $\square\ a = \frac{3}{4}$ did <u>not</u> get substituted into the expression by Simplify, which only rearranged in "descending order" the expression $\frac{1}{3}+a$ into $a+\frac{1}{3}$.

Beyond simple numerical calculations, Simplify will combine like terms.

$$\square(9x-3y)+(-2x+4y)$$
$$\triangle(9x-3y)+(-2x+4y)=7x+y \quad \textit{Simplify}$$

Simplify will not actually do multiplication, for example. Try to apply Simplify onto this expression:

$$\square\, x(2-x)$$
$$\triangle\, x(2-x)=x(-x+2) \quad \textit{Simplify}$$

All Simplify would do here is rearrange $2-x$ into $-x+2$ since that is descending order (variable term first, then constant), even though we humans would prefer it stay as $2-x$.

Simplify will also try to divide common terms away in fractions, like in these examples:

$$\square\frac{20}{6}$$
$$\triangle\frac{20}{6}=\frac{10}{3} \quad \textit{Simplify}$$
$$\square\frac{15}{4}\cdot\frac{12}{21}$$
$$\triangle\frac{15}{4}\cdot\frac{12}{21}=\frac{15}{7} \quad \textit{Simplify}$$
$$\square\frac{x-1}{x+3}\frac{x-5}{x-1}$$
$$\triangle\frac{x-1}{x+3}\frac{x-5}{x-1}=\frac{x-5}{x+3} \quad \textit{Simplify}$$

Simplify does not actually do the multiplications - that is left for Expand, nor any factoring, but rather Simplify looks for terms to divide away only, as you can see in this example:

$$\square\ \frac{x(x-1)}{x+3}\frac{x+3}{x-7}$$

$$\triangle\ \frac{x(x-1)}{x+3}\frac{x+3}{x-7} = \frac{x(x-1)}{x-7} \quad \textit{Simplify}$$

Simplify is not very smart about things either. One of the goals of *LiveMath* is not for the program to do the work for you, but rather for you to be intelligent human in charge of the math software, and for you to know what *LiveMath* needs. See this example of *LiveMath* being flummouxed on a simple minus sign issue:

$$\square\ \frac{x-1}{2-x}\frac{x-2}{1-x}$$

$$\triangle\ \frac{x-1}{2-x}\frac{x-2}{1-x} = \frac{(x-2)(x-1)}{(-x+2)(-x+1)} \quad \textit{Simplify}$$

Of course this entire expression should reduce to 1, if you factor out some -1 from the terms. *LiveMath* is just not going to do this for you without either some help from you. There are many ways to get *LiveMath* to do this, but they are beyond the scope of this chapter. The goal right now is for you to learn what *LiveMath does*, not what it cannot do.

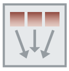
Simplify, strange as it may seem, is the trigger command for completing derivatives and integrals:

$$\square\ \frac{d}{dx}\left(x^3-9x^2+2x\right)$$

$$\triangle\ \frac{d}{dx}\left(x^3-9x^2+2x\right) = 3x^2-18x+2 \quad \textit{Simplify}$$

$$\square\ \int \sqrt[4]{x}\, dx$$

$$\triangle\ \int \sqrt[4]{x}\, dx = \tfrac{4}{5}x^{\frac{5}{4}} \quad \textit{Simplify}$$

You can see intermediate steps by turning off AutoSimplify: 

which is a toggle setting for either showing or hiding intermediate Simplify steps. Let's see the differentiation and integration Simplify commands again with AutoSimplify off:

$$\square \frac{d}{dx}\left(x^3-9x^2+2x\right)$$

$$\triangle \frac{d}{dx}\left(x^3-9x^2+2x\right)=3x^2-18x+2 \quad \textit{Simplify}$$

$$\square \int \sqrt[4]{x}\,dx$$

$$\triangle \int \sqrt[4]{x}\,dx = \frac{x\sqrt[4]{x}}{\frac{1}{4}+1} \quad \textit{Simplify}$$

The differentiation was the same, but the integral certainly had a less completed look: nearly, but not quite, the raw integration power function rule is being applied. If we used fractional exponent notation instead of the root notation, we would see it a bit better:

$$\square \int x^{\left(\frac{1}{4}\right)}\,dx$$

$$\triangle \int x^{\left(\frac{1}{4}\right)}\,dx = \frac{x^{\left(\frac{1}{4}\right)+1}}{\left(\frac{1}{4}\right)+1} \quad \textit{Simplify}$$

We will talk more about AutoSimplify as we move through this chapter. Toggling AutoSimplify on and off can be very useful for using *LiveMath* as a teaching tool, although the output is sometimes not ideal for this purpose. **AutoSimplify: ON** basically asks that the result be hit with the Simplify command - 10 times in fact - and the intermediate steps not shown.

3.2 Calculate 

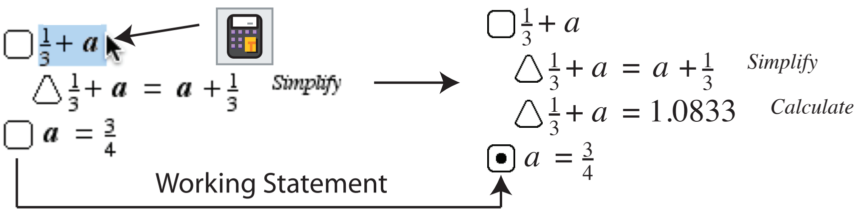
Calculate tries to make decimals. Calculate is more aggressive than Simplify, but has the same basic goal of an operation: to clean things up, but with a decimal answer as its goal.

If we start with our two fraction addition example, Calculate does not try to keep the answer as a nice fraction. In fact, Calculate converted both fractions to decimals immediately, and then added the decimals:

□ $\frac{1}{3}+\frac{3}{4}$

△ $\frac{1}{3}+\frac{3}{4} = 1.0833$ *Calculate*

If there is a variable in the expression, Calculate will scour the notebook for variable values in its attempts to come up with a decimal answer:



When *LiveMath* was asked to Calculate □ $\frac{1}{3}+a$ it converted the $\frac{1}{3} = 0.33333$, and then *LiveMath* tried to Calculate the variable a, which initiated a notebook search for a value of a. The first such Statement *LiveMath* found for a variable value assignment for a: □ $a = \frac{3}{4}$ *LiveMath* is telling you it found this statement value assignment for the variable a by placing a *hot dot* into the Assumption square to signify: "This value of variable a is being used elsewhere in the notebook".

While you may have multiple variable statements like this, only one can be used for the calculation. This is a larger topic about variable scope and Case Theories that will be discussed in detail in Chapter 6.

Only 1 definition of variable *a* is used for Calculate

□ $\frac{1}{3} + a$

△ $\frac{1}{3} + a = a + \frac{1}{3}$ *Simplify*

△ $\frac{1}{3} + a = 1.0833$ *Calculate*

⊡ $a = \frac{3}{4}$

□ $a = 5.1$

□ $a = \sqrt{2}$

Not used for Calculate

If Calculate cannot find a value for a variable, it will just do what it can to the rest of the expression:

□ $x + \frac{1.4}{7.3}$

△ $x + \frac{1.4}{7.3} = x + 0.19178$ *Calculate*

If Calculate cannot do anything meaningful to an expression, it will not give an output. If you calculate this expression:

□ $x + 2.1$

No output will be generated: *LiveMath* will try to create a new conclusion output triangle, but since the result is equal to the beginning assumption expression, it deletes the conclusion. That's why you will see a flash of a conclusion statement, then it is deleted.

The Calculate command is actually going on in a lot of computations and graphics inside of *LiveMath*, so understanding what it will do and not do will help you understand some higher program constructions.

3.3 Expand & Mini-Expand

Expand is the workhorse of symbolic computations in *LiveMath*. Expand is usually the command you want when you have an expression in *LiveMath*, and you want to whack it [1]: complete the multiplication and division operations in an expression. In some sense, Expand is the opposite of the Factor and Collect commands.

We can see best what Expand does by having AutoSimplify:OFF, so that the results from the Expand command are not followed with (a series of) Simplify. Let's see some examples:

$$\square\,(x+4)^3$$
$$\triangle\,(x+4)^3 = x^3+12x^2+48x+64 \quad \textit{Expand}$$

If we have a more complicated sub-expression, we can see where a single Expand command will stop:

$$\square\,([x+y]+4)^3$$
$$\triangle\,([x+y]+4)^3 = (x+y)^3+12(x+y)^2+48(x+y)+64 \quad \textit{Expand}$$

LiveMath did not fully multiply out these powers of (x+y) because AutoSimplify:OFF. If we turn AutoSimplify:ON, we see a different behavior:

AutoSimplify:OFF

$$\square\,([x+y]+4)^3$$
$$\triangle\,([x+y]+4)^3 = (x+y)^3+12(x+y)^2+48(x+y)+64 \quad \textit{Expand}$$
$$\triangle\,([x+y]+4)^3 = x^3+12x^2+48x+y^3+3xy^2+12y^2+3x^2y+24xy+48y+64 \quad \textit{Expand}$$

AutoSimplify:ON

With AutoSimplify:On, Expand is told to work with Simplify in a recursive approach – 10 recursions in fact – to thoroughly multiply out the expression. This demonstrates the power of AutoSimplify: Sometimes you want the expression completely whacked, sometimes you only want it partially whacked.

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1 In the older MathView program, the icon for Expand was a mallot *, the idea being that you wer going to whack an expression*

Expand will also complete division operations. With AutoSimplify:OFF we can look at this example:

$$\square \frac{x^3-6x^2+11x-6}{x-1}$$

$$\triangle \frac{x^3-6x^2+11x-6}{x-1} = \frac{-5x^2+11x-6}{x-1}+x^2 \quad \textit{Expand}$$

Expand did the polynomial division, with a remainder term. With AutoSimplify:OFF, Expand only did one recursion. We can hit the RHS again and again with Expand:

$$\square \frac{x^3-6x^2+11x-6}{x-1}$$

$$\triangle \frac{x^3-6x^2+11x-6}{x-1} = \frac{-5x^2+11x-6}{x-1}+x^2 \quad \textit{Expand}$$

$$\triangle \frac{x^3-6x^2+11x-6}{x-1} = \left(\frac{6x-6}{x-1}-5x\right)+x^2 \quad \textit{Expand}$$

$$\triangle \frac{x^3-6x^2+11x-6}{x-1} = \left(\left[6x^0\right]-5x\right)+x^2 \quad \textit{Expand}$$

$$\triangle \frac{x^3-6x^2+11x-6}{x-1} = x^2-5x+6 \quad \textit{Simplify}$$

We did a last Simplify to clean up the result. If we did the same calculation with AutoSimplify:ON, Expand and Simplify would have recursively hit this expression and achieved the final result in one output:

$$\square \frac{x^3-6x^2+11x-6}{x-1}$$

$$\triangle \frac{x^3-6x^2+11x-6}{x-1} = x^2-5x+6 \quad \textit{Expand}$$

MiniExpand is a lighter version of Expand. Sometimes you want to pulverize an expression, and sometimes you just want to tap it. See this example:

□ $\left(\left[x+2\right]^3+5\right)^2$

△ $\left(\left[x+2\right]^3+5\right)^2=\left(x^3+6x^2+12x+8\right)^2+10\left(x^3+6x^2+12x+8\right)+25$ *Expand*

△ $\left(\left[x+2\right]^3+5\right)^2=\left(x+2\right)^6+10\left(x+2\right)^3+25$ *MiniExpand*

MiniExpand is like an "Expand just on the outside" of an expression. It leaves alone the internal expressions from recursive Expand. These are both with AutoSimplify:OFF. With AutoSimplify:ON, the first Expand would have had the like terms combined:

□ $\left(\left[x+2\right]^3+5\right)^2$

△ $\left(\left[x+2\right]^3+5\right)^2=x^6+12x^5+60x^4+170x^3+300x^2+312x+169$ *Expand*

△ $\left(\left[x+2\right]^3+5\right)^2=\left(x+2\right)^6+10\left(x+2\right)^3+25$ *MiniExpand*

The MiniExpand was unaffected by the AutoSimplify change.

3.4 Factor & Collect

Factor and Collect are, in some senses, the opposite operations to Expand, at least for polynomials and integers, which is what Factor & Collect are mainly concerned with. Factor is a very powerful command, although it can be very time consuming and quite wonderous at times!

Factor works on two things: integers and polynomials. For integers, the expected factorization into primes will be the result:

□ 1400

△ $1400=2^3\cdot5^2\cdot7$ *Factor*

In similar fashion, Factor attempts to factorize polynomials, essentially the opposite operation to Expand. In this example, we apply select the RHS of the second line and click on the Factor button in the

Palette:

□ $(x+5)^2$

△ $(x+5)^2 = x^2+10x+25$ *Expand*

△ $(x+5)^2 = (x+5)^2$ *Factor*

For degree 2 polynomials, *LiveMath* will always stay symbolic, using the Quadratic Formula:

□ $7x^2+9x+1$

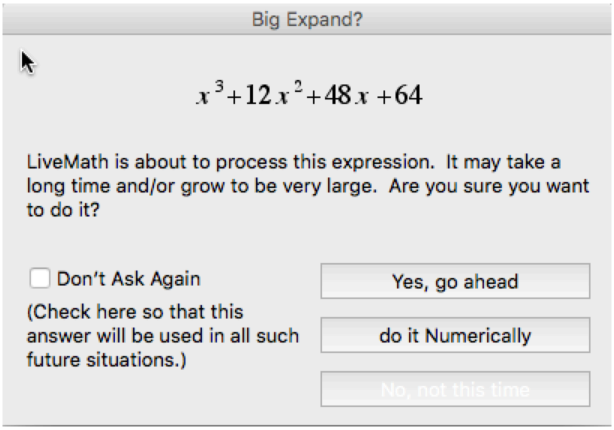
△ $7x^2+9x+1 = 7\left(x+\frac{1}{98}\sqrt{2597}+\frac{9}{14}\right)\left(x-\frac{1}{98}\sqrt{2597}+\frac{9}{14}\right)$ *Factor*

Imaginaries will be introduced if necessary:

□ $3x^2+9x+17$

△ $3x^2+9x+17 = 3\left(x+\frac{3}{2}+\frac{1}{6}\sqrt{123}\,i\right)\left(x+\frac{3}{2}-\frac{1}{6}\sqrt{123}\,i\right)$ *Factor*

If we try to Factor a polynomial of a slightly higher degree, say degree 3, we will often times get an intermediate dialog box asking us a question:



LiveMath was getting ready to freak out: similar to the Quadratic Formula, there is a general formula for the roots (or factor terms) for a degree 3 cubic, and for a degree 4 quartic, but these Cubic and Quartic formulas can grow in size and complexity and take a long time to compute. The option “Yes, go ahead” tells *LiveMath* to stay symbolic, and if *LiveMath* cannot find the polynomial factors easily (using other trickery, like testing for roots using divisors or the constant term), then it should go to the general cubic formula for the roots, messy as they are. *LiveMath* will do the same for degree 4 quartics.

$$\square (x+4)^3$$
$$\triangle (x+4)^3 = x^3+12x^2+48x+64 \quad \textit{Expand}$$
$$\triangle (x+4)^3 = (x+4)^3 \quad \textit{Factor}$$

In this case, *LiveMath* was able to crack the polynomial using other means before jumping into the Cubic formula. Here is an example of cubic polynomial that can only be cracked using the Cubic formula:

$$\square x^3+12x^2+48x+\pi$$
$$\triangle x^3+12x^2+48x+\pi = \left(x-\left[\tfrac{1}{3}\{-3\pi+576\}-128\right]^{\frac{1}{3}}+4\right)\left(x-\left[-\tfrac{1}{2}+\tfrac{1}{2}\sqrt{3}\,i\right]\left[\tfrac{1}{3}\{-3\pi+576\}-128\right]^{\frac{1}{3}}+4\right)\left(x-\left[-\tfrac{1}{2}-\tfrac{1}{2}\sqrt{3}\,i\right]\left[\tfrac{1}{3}\{-3\pi+576\}-128\right]^{\frac{1}{3}}+4\right) \quad \textit{Factor}$$

You may generate a similar example to show the Quartic formula at work.

LiveMath will not ask you this question for degree 5 quintics. Although there is a formula for the factors of a quintic and higher degree polynomials using *elliptic functions*, *LiveMath* is not going to go there. As many of us know, there is no formula for the factors of a quintic using roots and basic operations alone, the famous non-solvability of the quintic. It is incorrect to say “there is no formula for the roots of a quintic” - but rather the correct statement is “there is no formula for the roots of a quintic in terms of basic operations and roots”.

If you click on “do it Numerically”, or if you ask *LiveMath* to factor a degree 5 or higher, *LiveMath* will resort to some numerical routines for finding the factors of the polynomial, and these are a lot faster than the symbolic routines.

Sometimes the numerical factoring routines will find the easy integer factors in any case, but sometimes it will get consumed with numerical roundoff errors and such, and not get too far past the numerics, like in this example:

$$\square\; P = (x-1)^2(x-3)^2(x+5)(x-9)^2(x+11)$$
$$\triangle\; P = x^8 - 10x^7 - 114x^6 + 1454x^5 - 1280x^4 - 25278x^3 + 89298x^2 - 104166x + 40095 \quad \textit{Expand}$$
$$\triangle\; P = (x+11)(x+5)(x-1)^2(x-2.9999995)(x-3.0000005)(x-8.9999988)(x-9.0000012) \quad \textit{Factor}$$

We artificially greated a degree 8 polynomial with integer multiple roots, Expanded it, then Factored it. You can see *LiveMath* struggling with the numerics here, trying to show you these integer roots, as close as it can. *LiveMath* is limited to 15 decimal points of accuracy, a limitation that may be lifted in a future version.

We can see this problem in another example:

$$\square\; P = (x-1)^4(x-3)^3(x+11)$$
$$\triangle\; P = x^8 - 2x^7 - 74x^6 + 566x^5 - 1816x^4 + 3098x^3 - 2934x^2 + 1458x - 297 \quad \textit{Expand}$$
$$\triangle\; P = (x+11)(x-3)(x-3+2.4338563\times10^{-7}i)(x-3-2.4338563\times10^{-7}i)(x-1)^4 \quad \textit{Factor}$$

We know those roots aren't actually imaginary. *LiveMath* is doing its best to try to tell you "Hey, human in charge, I am getting really really close to that real root 3, you might have to help me!".

The Collect [icon] command is often confused with the less-precise mathematical language, "factor out the common terms". Since Factor is a very different function in *LiveMath*, the Collect command takes on this idea:

$$\square\; 3x^4 + 2x^2$$
$$\triangle\; 3x^4 + 2x^2 = 3x^2\left(x - \tfrac{1}{3}\sqrt{6}\,i\right)\left(x + \tfrac{1}{3}\sqrt{6}\,i\right) \quad \textit{Factor}$$
$$\triangle\; 3x^4 + 2x^2 = x^2\left(3x^2 + 2\right) \quad \textit{Collect}$$

When we hit this polynomial $3x^4 + 2x^2$ with Factor, then Factor does what it does: it hammers it into submission, complex factors and all. But as mathematicians we see very easily that we really just want the common term x^2 "factored out" of both terms. In *LiveMath* language, we will Collect out the common factor x^2 from both terms.

Collect works on more than just polynomials, so you can see that it is really a very different manipulation than is Factor.

$$\square\ \sin(x)\sin(y)+3\sin(y)\tan(x)$$
$$\triangle\ \sin(x)\sin(y)+3\sin(y)\tan(x)=(\sin[x]+3\tan[x])\sin(y) \quad \textit{Collect}$$

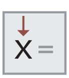
Of course, Factor wouldn't touch this expression, since Factor only works on polynomials. If you try Factor on this example:

$$\square(\sin[x])^2-5\sin(x)+6$$
$$\triangle(\sin[x])^2-5\sin(x)+6=\left(i\sqrt{-5\sin[x]+6}+\sin[x]\right)\left(-i\sqrt{-5\sin[x]+6}+\sin[x]\right) \quad \textit{Factor}$$

Factor is not at all happy, and neither are you. You might think that "*LiveMath* should just know what I want here", but that will not bring you happiness. *LiveMath* is just a tool, and part of its beauty (and usefulness in teaching) is that it does not do everything for you. So to Factor this express, you first need to get the expression into a polynomial of a variable, factor that new polynomial, then reverse substitute:

$$\square(\sin[x])^2-5\sin(x)+6$$
$$\triangle(\sin[x])^2-5\sin(x)+6=y^2-5y+6 \quad \textit{Substitute}$$
$$\triangle(\sin[x])^2-5\sin(x)+6=(y-3)(y-2) \quad \textit{Factor}$$
$$\triangle(\sin[x])^2-5\sin(x)+6=(\sin[x]-3)(\sin[x]-2) \quad \textit{Substitute}$$
$$\square\ \sin(x)=y$$
$$\triangle\ y=\sin(x) \quad \textit{Isolate}$$

This is a nice segue into the next topic in our hit parade.

3.5 Isolate 

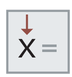
In higher programs like *Mathematica*, there are very powerful commands like `Solve[]` and `NSolve[]` – workhorse commands that use the most powerful symbolic and numerical recipes to solve equations (and systems of equations) both large and small, to bring any equation that has a solution into submission. Hit the equation with `Solve[]`, and your "black box" answer is delivered to you in seconds (and sometimes it takes a Ph.D. to interpret these answers from *Mathematica*!), but with no inkling as to what this symbolic goliaths actually did to get at that answer.

LiveMath has no such *solve* feature.

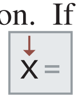
LiveMath makes you work for your solutions to equations, even easy ones. Remember that *LiveMath*'s goal is to emulate how you do computation on paper, manually. So if you have a very messy and complicated equation that makes you scratch your head and say, "Wow, I have no idea how I would ever solve that by hand", and you just want a solution (no more talking), then reach into your tool box, get *Mathematica* or *Maple* or *Matlab* or *R* something similar, and ask those higher programs to grind out the solution you seek.

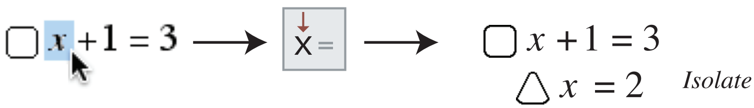
We belabor this point because nearly every (impatient) novice *LiveMath* user invariably says, "Where the heck is the SOLVE command?"

There isn't one. Stop looking for it.

The closest thing to a Solve command is: Isolate 

Often times, *Isolate* is what you mean when you say "solve this equation for x".

Let's look at a first easy example: Suppose we wish to solve the equation $x + 1 = 3$ for x. What we really want to do is isolate the x variable: to do a series of algebraic operations to achieve a new equation with $x =$ something that solves the original equation. If we select the x variable, and then click on the Palette button  to invoke the Isolate command, we will get this:



There are multiple ways to achieve the same output in *LiveMath*, and more instructional ways as we will see. One other way to invoke the Isolate command is to select the *x* variable in the equation, then hold down the ⌘ or ctrl key, and drag the *x* to the square Assumption icon:

For more a more in-depth discussion of this kind of action, see Chapter 6 on Selecting & Drag & Drop.

We can see more of the intermediate operations with Isolate if we flick AutoSimplify:OFF:

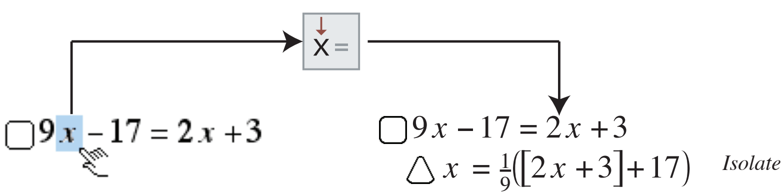
□ $x+1=3$

△ $x=3-1$ *Isolate*

△ $x=2$ *Simplify*

We see here what Isolate is really doing: to isolate the x variable, it "unwrapped" the operations near *x*, getting that "+1" to the other side of the equation, converting it to its opposite operation: "-1".

If you try an Isolate on an equation with multiple instances of the variable x, you will get what you asked for: isolation of the selected variable:

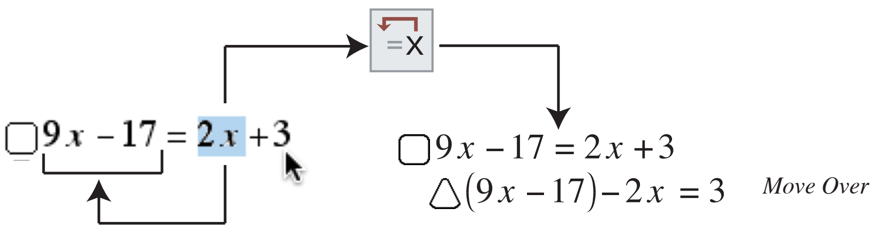


Don't get mad at LiveMath here! LiveMath did exactly what you asked. You highlighted the *x* variable on the left, and did an Isolate command, so LiveMath isolated that selected *x*!

Just like on paper with manual calculations, you have to do some algebra to get "all of the x variables together" as we teach in high school algebra. In LiveMath, you have to do the same thing. We need a command to <u>move</u> <u>over</u> the terms from one side of the equation to the other. Thus, our next section.

3.6 MoveOver

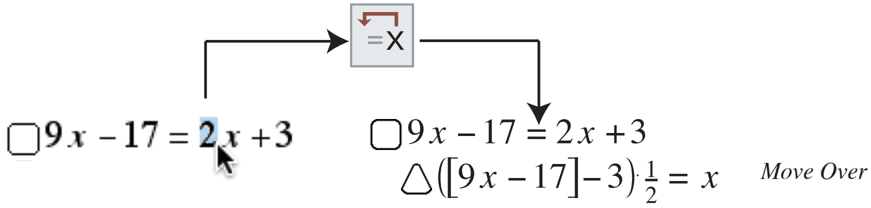
Utilizing the working example from the last section, we will use the MoveOver command to "algebraically move" the *x* variable term on the RHS to the LHS of the equation:



AutoSimplify:OFF allows us to see the raw effect of the MoveOver command. If you changed to AutoSimplify:ON, then the MoveOver would be combined with some Simplify commands:

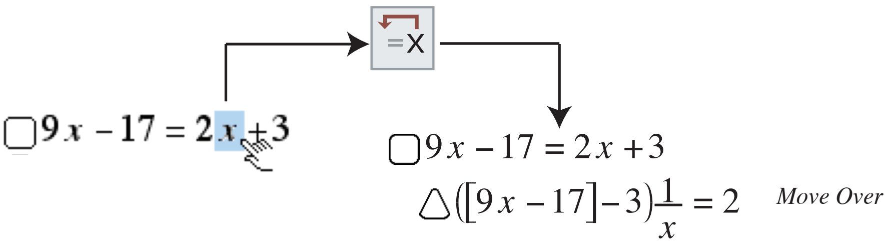
$$9x - 17 = 2x + 3$$
$$(9x - 17) - 2x = 3 \quad \textit{Move Over}$$
$$7x - 17 = 3 \quad \textit{Move Over}$$

You can MoveOver anything in the equation. Let's try moving over the "2" from the RHS to the LHS:

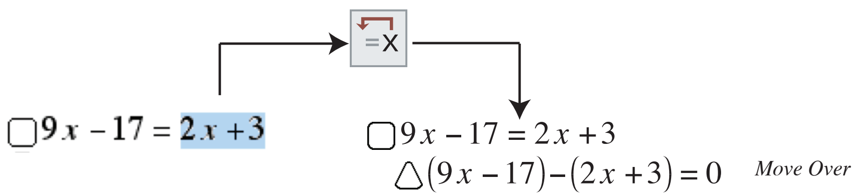


Notice that LiveMath had to first MoveOver the "+3" from the RHS to the LHS first, before doing the second MoveOver on the "•2". MoveOver is about unwrapping operations, in reverse order of operations order.

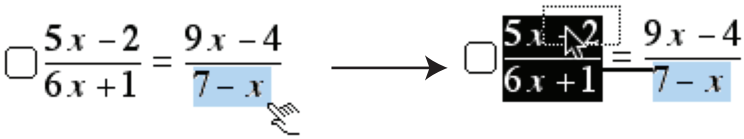
LiveMath will MoveOver whatever you wish. If we want to MoveOver the x variable from the RHS to the LHS, LiveMath will do it, even though it is not really mathematically advantageous to do so.



You can select the entire RHS and move it to the LHS:



In expressions involving division, the MoveOver command is exactly the way you think of *cross multiplication*:



$$\square\ \frac{5x-2}{6x+1}=\frac{9x-4}{7-x}$$

$$\triangle\ \frac{5x-2}{6x+1}(7-x)=9x-4 \qquad \textit{Move Over}$$

And then finishing this off by doing a MoveOver on $6x+1$ to the RHS:

$$\square\ \frac{5x-2}{6x+1}=\frac{9x-4}{7-x}$$

$$\triangle\ \frac{5x-2}{6x+1}(7-x)=9x-4 \qquad \textit{Move Over}$$

$$\triangle\ (-x+7)(5x-2)=(9x-4)(6x+1) \qquad \textit{Move Over}$$

As a nice summary of all of the commands we have learned about so far, here is a set of operations to get at the solutions of this equation:

$$\square\ \frac{5x-2}{6x+1}=\frac{9x-4}{7-x}$$

$$\triangle\ \frac{5x-2}{6x+1}(7-x)=9x-4 \qquad \textit{Move Over}$$

$$\triangle\ (-x+7)(5x-2)=(9x-4)(6x+1) \qquad \textit{Move Over}$$

$$\triangle\ -5x^2+37x-14=54x^2-15x-4 \qquad \textit{Expand}$$

$$\triangle\ -59x^2+52x-10=0 \qquad \textit{Move Over}$$

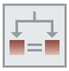
$$\triangle\ -59\left(x+\tfrac{1}{3481}\sqrt{299366}-\tfrac{26}{59}\right)\left(x-\tfrac{1}{3481}\sqrt{299366}-\tfrac{26}{59}\right)=0 \qquad \textit{Factor}$$

$$\triangle\ x=-\tfrac{1}{3481}\sqrt{299366}+\tfrac{26}{59} \qquad \textit{Isolate}$$

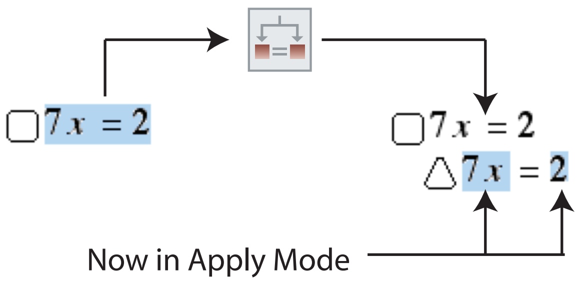
$$\triangle\ x=0.2835 \qquad \textit{Calculate}$$

$$\triangle\ x=\tfrac{1}{3481}\sqrt{299366}+\tfrac{26}{59} \qquad \textit{Isolate}$$

$$\triangle\ x=0.59786 \qquad \textit{Calculate}$$

3.7 Apply 

Apply is a command custom built for teaching. The idea is that you "apply same operations to both sides of equality and preserve equality". Let's see an example:



After the Apply click, the conclusion statement is in a special mode: Apply Mode. Whatever you enter in (via Palette or the keyboard or the Insert Menu) will be applied to both sides of the equation, in tandem.

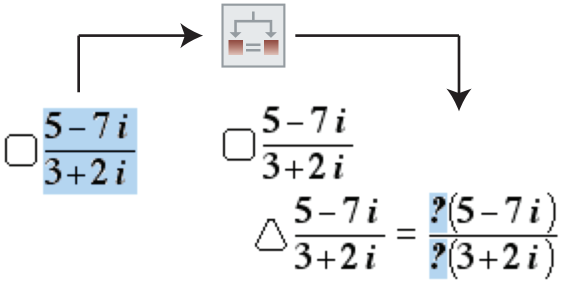
So let's type in: / 7

$$\square\ 7x = 2$$
$$\triangle\ \frac{7x}{7} = \frac{2}{7} \quad \textit{Apply}$$

To end Apply Mode, click anywhere in the notebook window: you will hear a beep and the *Apply* command description will show, signifying that the Apply command has concluded.

As you can see, the Apply command is very helping in instructional situations for teaching students about balancing an equation with operations.

You can use the Apply command on fractions as well, balancing the numerator and denominator with equal operations. In this example, we start with a fraction of complex numbers, and we want to complete the division by multiplying numerator and denominator by the complex conjugate:

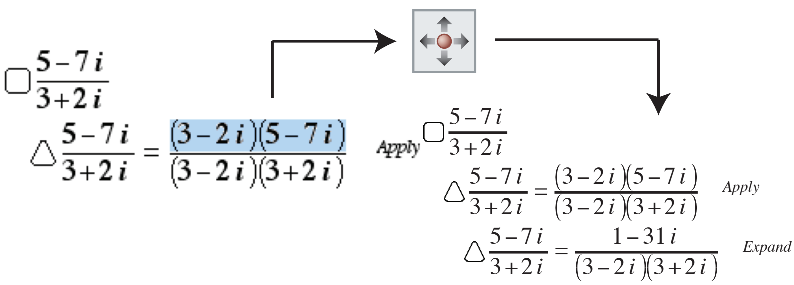


Now in Apply Mode we type: [(] [3] [-] [2] [I] [)] and then click in the notebook window to end Apply Mode:

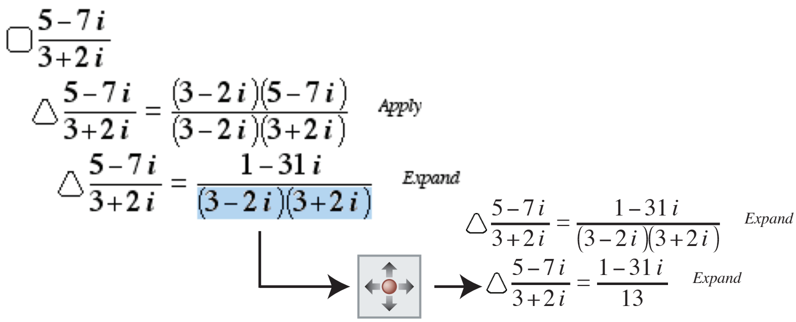
$$\frac{5-7i}{3+2i}$$

$$\frac{5-7i}{3+2i} = \frac{(3-2i)(5-7i)}{(3-2i)(3+2i)} \quad \textit{Apply}$$

Then we have select the numerator and denominator each (seperately) and do an Expand to complete the multiplications:



And then do the same to the denominator:



One more Expand the entire RHS expression finishes up the computation:

$$\frac{5-7i}{3+2i}$$

$$\frac{5-7i}{3+2i}=\frac{(3-2i)(5-7i)}{(3-2i)(3+2i)} \quad \textit{Apply}$$

$$\frac{5-7i}{3+2i}=\frac{1-31i}{(3-2i)(3+2i)} \quad \textit{Expand}$$

$$\frac{5-7i}{3+2i}=\frac{1-31i}{13} \quad \textit{Expand}$$

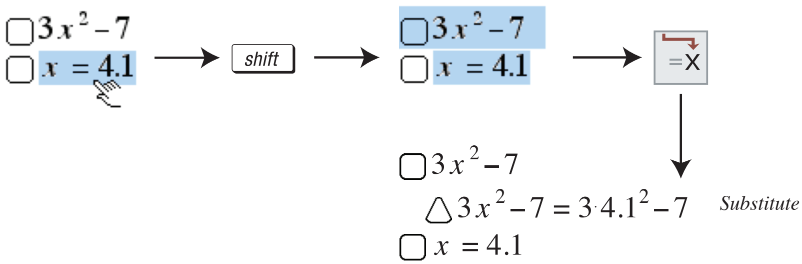
$$\frac{5-7i}{3+2i}=\frac{1}{13}-\frac{31}{13}i \quad \textit{Expand}$$

3.8 Substitute

We have left the most beautiful and powerful command in the *LiveMath* arsenal for (nearly) last: Substitute.

There is a Palette button for Substitute, but you won't use it much, favoring the drag&drop version of the command. We will show it to you anyway, just for completeness.

The operation comes in 2 steps, and the order of selection is important. First you select what you *want* to substitute; then, hold down the shift key, and then select *where* you want to substitute:



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